

Landau confining replica model from an explicitly breaking of a $SU(3)$ group without auxiliary fields.

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Abstract

We propose a mechanism displaying gluon confinement, as defined by the behavior of the propagators, in a model of $SU(2)$ gauge fields. The model originates from an explicitly broken $SU(3)$ gauge theory giving rise to a replica model composed of three mixed $SU(2)$ groups. The mechanism consists in the usual $SU(3)$ Yang-Mills theory in the Landau gauge, with a soft breaking term in such a way as to change the field propagation and group content at low energies. The relation of this soft mass term with the Gribov problem is presented and the link between soft terms and the scaling and decoupling solutions is discussed.

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1 Introduction

The problem of gluon confinement is the core of the general investigations of strongly coupled gauge theories. An important aspect of gluon confinement is related to the behavior of the gluon propagator.

Recently this problem has received great attention in different approaches. One of these approaches comes from lattice simulation where the behavior of the gluon propagator in the infrared regime is studied [1, 28, 3]. These results display positivity violation thus making impossible a particle interpretation for the gluon excitation at low energies. This is taken as a strong signal of gluon confinement. In the analytical point of view, one possible approach of the confinement problem comes from the analysis of the Gribov copies [4], where the Gribov-Zwanziger (GZ) model [5, 26], and this refined version, the so-called Refined Gribov-Zwanziger (RGZ) model [7], take place. Also, a recently developed model based on the introduction of a replica of the Faddeev-Popov action enjoys a confined gluon propagator¹ [8]. Usually, these models provide propagators behaving as²

$$D(p^2) = \frac{p^2}{p^4 + \gamma^4}, \quad (1)$$

where $D(p^2)$ is the gluon form factor in Euclidean spacetime and γ is a mass parameter³. It is easy to notice that a propagator of this type has complex poles, being impossible its identification with a propagation of a physical particle. In other words, it is a suitable candidate to be a confined object.

An important feature that has been studied about propagators like (1) is that they can be seen as a propagation of two unphysical modes with imaginary squared masses $\pm i\gamma^2$, named *i*-particles [9]. In fact, one can immediately notice that the form factor (1) can be written as

$$\frac{p^2}{p^4 + \gamma^4} = \frac{1}{2} \left(\frac{1}{p^2 - i\gamma^2} + \frac{1}{p^2 + i\gamma^2} \right). \quad (2)$$

Despite the fact that such propagators do not have an interpretation in the physical spectrum, it is still possible to construct composite operators, $\mathcal{O}[A]$, whose the correlation functions exhibit a Källén-Lehmann spectral representation:

$$\langle \mathcal{O}(p) \mathcal{O}(-p) \rangle = \int_{\tau_0}^{\infty} d\tau \frac{\rho(\tau)}{\tau + p^2}, \quad (3)$$

where $\rho(\tau)$ is the positive spectral density and $\tau_0 \geq 0$ stands for the threshold. An important feature of expression (3) is that we can move from Euclidean to Minkowski space. Moreover, the positivity of the spectral density $\rho(\tau)$ enables us to give an interpretation of (3) in terms of physical states with positive norm.

¹We will refer to this model just as *the replica model*.

²In the case of the RGZ model the gluon propagator take into account the effects of dimension two condensates and it is more complex. Typically, the gluon form factor looks like

$$D(p^2) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + (\gamma^4 + M^2 m^2)},$$

where M and m are mass parameters associated with the condensates, while γ is the so-called Gribov mass parameter.

³In the GZ model this parameter is known as the Gribov parameter, which is directly associated with the restriction of the Feynman path integrals to the Gribov region.

In the present work, we will study a $SU(3)$ Euclidean Yang-Mills theory and try to explore a well known property of this group, which consists in the fact that it contains three mixed $SU(2)$ groups [10, 11]. In the presence of a dimension two condensate, $\langle d^{abc} A_\mu^b A_\mu^c \rangle$, with d^{abc} being the totally symmetric structure constant of the $SU(3)$ Lie algebra, the color group symmetry is explicitly broken. However, two of these three $SU(2)$ groups are related in a structure of i -particles, and, thanks to a residual symmetry, it is possible to write composite operators having a spectral representation like (3).

The paper is organized as follows. In Sect. 2, we make a brief review about the GZ model, the introduction of the i -particles, and the replica model. In Sect. 3, we explain how the three mixed $SU(2)$ groups can emerge from the $SU(3)$. In Sect. 4, we introduce the condensate $\langle d^{abc} A_\mu^b A_\mu^c \rangle$ and write the action corresponding to the model that we desire to study. Also in this section we show how this condensate modifies the gluon propagator and how the i -particle structure appears between two of the three $SU(2)$ groups. In Sect. 5, the relation of the mass terms, obtained from the introduction of dimension two condensates, and the scaling and decoupling solution is established. The Sect. 6 is dedicated to discuss the Källén-Lehmann spectral representation of the candidates to be a physical observable. Our conclusions are presented in Sect. 7. The symmetry content of the model, characterized by a full set of Ward identities compatible with the Quantum Principle Action [12], and the proof of its renormalizability are presented in the Appendix A.

2 A brief review: the GZ model, i -particles and the replica model

2.1 The GZ model

In order to clarify the understanding of the Gribov problem and its correlation with our proposal we will present below a description of the problem in the Landau gauge. The Euclidean $SU(N)$ Yang-Mills action in the Landau gauge is given by:

$$S_{\text{YM}} = \int d^4x \left(\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + ib^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b \right), \quad (4)$$

where

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c \quad (5)$$

and

$$D_\mu^{ab} = \delta^{ab} \partial_\mu - gf^{abc} A_\mu^c. \quad (6)$$

Here, A_μ^a is the gauge field, b^a is a Lagrange multiplier enforcing the Landau gauge, $\partial_\mu A_\mu^a = 0$, (\bar{c}^a, c^a) are a pair of anti-commuting scalar fields known as the Faddeev-Popov ghost fields, and g is the coupling constant of the theory. The labels (a, b, c, \dots) run to 1 to $(N^2 - 1)$ and f^{abc} are the totally anti-symmetric structure constant of the Lie algebra of the generator of $SU(N)$. Also, This action is left invariant under the following nilpotent BRST transformations:

$$sA_\mu^a = -D_\mu^{ab} c^b, \quad sc^a = \frac{g}{2} f^{abc} c^b c^c, \quad s\bar{c}^a = ib^a, \quad sb^a = 0. \quad (7)$$

Although the gauge be fixed by the Faddeev-Popov method, Gribov showed in [4]⁴ that there are still field configurations obeying the Landau gauge linked by gauge transformations, *i.e.* there are still equivalent configurations, or copies, being taken into account into the Feynman path integral. In other words, the

⁴See also [13] for a pedagogical review.

gauge is not completely fixed and the remaining ambiguity is allowed due to the existence of normalizable zero-modes of the Faddeev-Popov operator,

$$\mathcal{M}^{ab} = -\partial_\mu D_\mu^{ab}. \quad (8)$$

Gribov also showed that to eliminate these copies the domain of integration of the functional integral should be restricted to a certain region Ω , the so-called Gribov region, that is defined as the set of field configurations performing the Landau gauge condition, for which the Faddeev-Popov operator is strictly positive, namely

$$\Omega := \{ A_\mu^a \mid \partial_\mu A_\mu^a = 0, \mathcal{M}^{ab}(A) > 0 \}. \quad (9)$$

Its boundary, $\partial\Omega$, where the first vanishing eigenvalue of the Faddeev-Popov operator shows up, is known as the Gribov horizon.

As in the region Ω the Faddeev-Popov operator is positive than its inverse must diverge when approaching the horizon, due to the existence of a zero mode. So the restriction to the first Gribov region is implemented requiring that

$$G(p, A) = \frac{\delta^{ab}}{N^2 - 1} \langle p | (-\partial_\mu D_\mu^{ab})^{-1} | p \rangle, \quad (10)$$

which is the normalized trace of the ghost connected two point function in momentum space, has no pole for a given nonvanishing value of the momentum p , except for the singularity at $p = 0$, corresponding to the first Gribov horizon. At $p \approx 0$ one can write

$$\begin{aligned} G(p, A) &\approx \frac{1}{p^2} \frac{1}{1 - \sigma(p, A)}, \\ \sigma(p, A) &= \frac{N}{N^2 - 1} \frac{1}{p^2} \int \frac{d^4 q}{(2\pi)^4} \frac{(p - q)_\mu p_\nu}{(p - q)^2} A_\mu^a(-q) A_\nu^a(q). \end{aligned} \quad (11)$$

From the above expression (11), it follows that the no-pole condition at finite nonvanishing p is

$$\sigma(p, A) < 1. \quad (12)$$

As $\sigma(p, A)$ decreases as p^2 increases one can also take

$$\sigma(0, A) = \frac{1}{4} \frac{N}{N^2 - 1} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} (A_\mu^a(-p) A_\mu^a(p)) \leq 1. \quad (13)$$

It is important to emphasize here that we work with the trace of the ghost propagator to find the restriction to the Gribov region. This is a particularity of the Gribov mechanism in the Landau gauge. This is related to the convexity of the Gribov region in the Landau gauge. Other gauges, like the maximal Abelian gauge, does not necessarily present the same property [14, 15, 16]⁵.

In order to perform the restriction to the Gribov region into the partition function, \mathcal{Z} , the final step is to introduce the no-pole condition with the help of a Heaviside function:

$$\mathcal{Z} = \int \mathcal{D}A \delta(\partial A) \theta(1 - \sigma(0, A)) \exp^{-S_{\text{YM}}}. \quad (14)$$

This will give rise to a propagator for the gauge field of the type

$$\langle A_\mu^a(-p) A_\nu^b(p) \rangle = \delta^{ab} \frac{p^2}{p^4 + \gamma^4} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right), \quad (15)$$

⁵In the maximal Abelian gauge we take only the trace of diagonal ghost propagator.

and the Gribov parameter γ is fixed by the gap equation:

$$\frac{3Ng^2}{4} \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^4 + \gamma^4} = 1. \quad (16)$$

It is clear that the Gribov approach is only the first step in order to consistently treat the problem of zero modes and the Gribov copies in a gauge fixed Yang-Mills theory. The second step is the GZ theory [5, 26], which consists in a renormalizable and local way to implement the restriction to the first Gribov region. In fact, Zwanziger observed that the restriction could be implemented by adding the following term in the action (4):

$$S_{\text{GZ}} = S_{\text{YM}} + \gamma^4 H(A), \quad (17)$$

where, $H(A)$ is the so-called horizon function,

$$H(A) = g^2 \int d^4x d^4y f^{abc} A_\mu^b(x) [\mathcal{M}^{-1}]^{ad}(x, y) f^{dec} A_\mu^e. \quad (18)$$

In the Zwanziger approach, the parameter γ is fixed by the equation

$$\langle H(A) \rangle = 4V(N^2 - 1), \quad (19)$$

where V is the Euclidean space volume. Notice that the Gribov form factor (13) coincides with the first order of the horizon function⁶:

$$\frac{H(A)}{4V(N^2 - 1)} = \sigma(0, A) + O(A^3). \quad (20)$$

It is clear that the horizon function is nonlocal, but it can be localized with the help of a suitable set of auxiliary fields. In order to ensure that those extra fields do not introduce extra degrees of freedom they are introduced in the form of a BRST quartet⁷:

$$\begin{aligned} s\bar{\omega}_\mu^{ab} &= \bar{\varphi}_\mu^{ab}, & s\bar{\varphi}_\mu^{ab} &= 0, \\ s\varphi_\mu^{ab} &= \omega_\mu^{ab}, & s\omega_\mu^{ab} &= 0, \end{aligned} \quad (21)$$

where $(\bar{\varphi}, \varphi)$ are a pair of complex commuting fields, while $(\bar{\omega}, \omega)$ are anti-commuting ones. Now, the local version of the GZ action is then given by:

$$S_{\text{GZ}}^{\text{local}} = S_{\text{YM}} + \int d^4x \left[\bar{\varphi}_\mu^{ac} \mathcal{M}^{ab} \varphi_\mu^{bc} - \bar{\omega}_\mu^{ac} \mathcal{M}^{ab} \omega_\mu^{bc} + \gamma^2 g f^{abc} (\varphi_\mu^{ab} - \bar{\varphi}_\mu^{ab}) A_\mu^c \right]. \quad (22)$$

It is quite easy to notice that this term explicitly breaks the BRST symmetry. Then, following the Zwanziger steps, in order to establish a local, renormalizable and BRST-invariant theory, we define a most general invariant action, which possesses action (22) as a particular physical case. Such desired action is then given by:

$$\begin{aligned} S_{\text{GZ}}^{\text{local-inv}} &= S_{\text{YM}} + s \int d^4x \left[\bar{\omega}_\mu^{ac} \mathcal{M}^{ab} \varphi_\mu^{bc} + \bar{Q}_{\mu\nu}^{ab} D_\mu^{ac} \varphi_\nu^{cb} + J_{\mu\nu}^{ab} D_\mu^{ac} \bar{\omega}_\nu^{cb} + \bar{Q}_{\mu\nu}^{ab} J_{\mu\nu}^{ab} \right] \\ &= S_{\text{YM}} + \int d^4x \left\{ \bar{\varphi}_\mu^{ac} \mathcal{M}^{ab} \varphi_\mu^{bc} - \bar{\omega}_\mu^{ac} \mathcal{M}^{ab} \omega_\mu^{bc} + f^{abc} (\partial_\mu \omega_\nu^{ae}) (D_\mu^{bd} c^d) \varphi_\nu^{ce} \right. \\ &\quad + \bar{J}_{\mu\nu}^{ab} D_\mu^{ac} \varphi_\nu^{cb} - \bar{Q}_{\mu\nu}^{ab} \left[D_\mu^{ac} \omega_\nu^{cb} + g f^{acd} (D_\mu^{bd} c^d) \varphi_\nu^{cb} \right] + J_{\mu\nu}^{ab} \left[D_\mu^{ac} \bar{\varphi}_\nu^{cb} + g f^{acd} (D_\mu^{bd} c^d) \bar{\omega}_\nu^{cb} \right] \\ &\quad \left. + \bar{Q}_{\mu\nu}^{ab} D_\mu^{ac} \bar{\omega}_\nu^{cb} + \left(\bar{J}_{\mu\nu}^{ab} J_{\mu\nu}^{ab} - \bar{Q}_{\mu\nu}^{ab} Q_{\mu\nu}^{ab} \right) \right\}, \end{aligned} \quad (23)$$

⁶Actually, the form factor can be calculated at all orders and the result is that such coincidence occurs in fact at all orders. In [17], this equivalence is proved at third order, and in [18] it is proved at all orders.

⁷Actually, the BRST quartet is composed by two BRST doublets, which has the basic structure

$$sU = V, \quad sV = 0,$$

for a generic pair of fields (U, V) , guarantying the nilpotence of the BRST operator, s , *i.e.* $s^2 = 0$.

where, the set of external sources

$$\{\bar{J}_{\mu\nu}^{ab}, J_{\mu\nu}^{ab}, \bar{Q}_{\mu\nu}^{ab}, Q_{\mu\nu}^{ab}\} \quad (24)$$

forms a BRST quartet structure, *i.e.*

$$\begin{aligned} s\bar{Q}_{\mu\nu}^{ab} &= \bar{J}_{\mu\nu}^{ab}, & s\bar{J}_{\mu\nu}^{ab} &= 0, \\ sJ_{\mu\nu}^{ab} &= Q_{\mu\nu}^{ab}, & sQ_{\mu\nu}^{ab} &= 0, \end{aligned} \quad (25)$$

being (\bar{J}, J) a pair of commuting sources and (\bar{Q}, Q) a pair of anti-commuting ones. The last term is a vacuum term permitted by power-counting and it is necessary to obtain the gap equation (19) by demanding that the vacuum energy, \mathcal{E} , is independent of γ^2 , *i.e.*

$$\frac{\partial \mathcal{E}}{\partial \gamma^2} = 0. \quad (26)$$

The original action (22) can be recovered from (23) when these external sources attain their *physical values*. Namely,

$$\bar{J}_{\mu\nu}^{ab} \Big|_{phys} = -\bar{J}_{\mu\nu}^{ab} \Big|_{phys} = \gamma^2 \delta^{ab} \delta_{\mu\nu}, \quad \bar{Q}_{\mu\nu}^{ab} \Big|_{phys} = \bar{Q}_{\mu\nu}^{ab} \Big|_{phys} = 0. \quad (27)$$

After perform a linear shift on the ω_μ^{ab} variable,

$$\omega_\mu^{ac} \rightarrow \omega_\mu^{ac} - (\mathcal{M}^{-1})^{ab} \left[\partial_\nu \left(g f^{bde} \varphi_\mu^{dc} D_\nu^{ef} c^f \right) - \gamma^2 g f^{bdc} D_\mu^{de} c^e \right], \quad (28)$$

one can show that

$$\left(S_{GZ}^{local-inv} \right) \Big|_{phys} \equiv S_{GZ}^{local}. \quad (29)$$

It is important to emphasize here that the Zwanziger approach described above has received some improvements in recent years. In [19, 20], the model was formulated in such way that the BRST symmetry breaking appears as a linear breaking, while in [21], the breaking appears as a spontaneous breaking, instead of an explicit one⁸.

2.2 The *i*-particles

As already seen, eq. (2) suggests that a theory presenting Gribov-like propagators, can be rewritten in terms of *i*-particles, *i.e.* with propagations of “particles” with complex squared masses. Now, let us take a look on this concept following the lines outlined in [9]. Then, we will start with a scalar field toy model exhibiting a confining Gribov-type propagator:

$$S = \int d^4x \frac{1}{2} \Phi \left(-\partial^2 + \frac{\theta^4}{-\partial^2} \right) \Phi, \quad (30)$$

where θ is a mass parameter playing an analogous role of the Gribov parameter γ . The resulting propagator is the desired Gribov-type:

$$\langle \Phi(p) \Phi(-p) \rangle = \frac{p^2}{p^4 + \theta^2}, \quad (31)$$

and it can be cast in a local form exactly like in the case of the GZ model:

$$S = \int d^4x \left[\frac{1}{2} \Phi(-\partial^2) \Phi + \bar{\varphi}(-\partial^2) \varphi + \sqrt{2} \theta^2 \Phi(\varphi - \bar{\varphi}) - \bar{\omega}(-\partial^2) \omega \right]. \quad (32)$$

⁸The renormalization of this formulation was already proven in [22].

As $(\bar{\varphi}, \varphi)$ form a pair of complex field we can decouple the real part from the theory. In fact, defining

$$\varphi = \frac{1}{\sqrt{2}}(U + iV), \quad \bar{\varphi} = \frac{1}{\sqrt{2}}(U - iV), \quad (33)$$

one can write

$$S = \int d^4x \left[\frac{1}{2} \Phi(-\partial^2) \Phi + \frac{1}{2} V(-\partial^2) V + 2i\theta^2 \Phi V - \bar{\omega}(-\partial^2) \omega + \frac{1}{2} U(-\partial^2) U \right]. \quad (34)$$

From now on, we will neglect the decoupled fields $(U, \bar{\omega}, \omega)$ and we will diagonalize the action above by introducing the new field variables:

$$\Phi = \frac{1}{\sqrt{2}}(\lambda + \eta), \quad V = \frac{1}{\sqrt{2}}(\lambda - \eta). \quad (35)$$

Thus, we have

$$S = \int d^4x \left[\frac{1}{2} \lambda(-\partial^2 + 2i\theta^2) \lambda + \frac{1}{2} \eta(-\partial^2 - 2i\theta^2) \eta \right], \quad (36)$$

and the propagators in terms of the new field variables stand by

$$\langle \lambda(p) \lambda(-p) \rangle = \frac{1}{p^2 + 2i\theta^2}, \quad \langle \eta(p) \eta(-p) \rangle = \frac{1}{p^2 - 2i\theta^2}. \quad (37)$$

From this expression one immediately sees that the fields λ and η correspond to the propagation of unphysical modes with complex squared masses $\pm 2i\theta$. These are the so-called i -particles of the model. Notice also that, despite the imaginary terms, the action (37) is Hermitian if we require that $\lambda^\dagger = \eta$.

We already argued that the excitations in terms of i -particles are unphysical. Nevertheless, following [9], physical states can be introduced by constructing suitable composite operators out of the fields (λ, η) which exhibit desirable analyticity properties, as encoded in the Källén-Lehmann spectral representation. Such composite operators are obtained by requiring that the i -particles fields enter in pairs, *i.e.* the desired operator contains as many fields of the type λ as of the type η . It ensures that in the corresponding correlation function only complex conjugate pairs of i -particles propagate in the Feynman diagrams. In the present case, the simplest example of a local composite operator with the required physical properties is $\mathcal{O}(x) = \lambda(x)\eta(x)$. In [9] it was shown that the correlation function $\langle \mathcal{O}(p)\mathcal{O}(-p) \rangle$ has a well defined Källén-Lehmann spectral representation⁹:

$$\langle \mathcal{O}(p)\mathcal{O}(-p) \rangle = \int \frac{d^4q}{(2\pi)^4} \frac{1}{(p-q)^2 - 2i\theta^2} \frac{1}{q^2 + 2i\theta^2} \equiv \int_{\tau_0}^{\infty} d\tau \frac{\rho(\tau)}{\tau + p^2}, \quad (38)$$

with

$$\rho(\tau) = \frac{1}{(4\pi)^2} \sqrt{1 - \frac{16\theta^4}{\tau^2}}, \quad \tau_0 = 2\theta^2. \quad (39)$$

Furthermore, a model with interacting i -particles is dealt in [23].

⁹Here we present only the $4d$ result. The results for the spectral density in $2d$ and $3d$ can also be found in [9].

2.3 The replica model

Let us now describe, in few words, the so-called replica model. It was first introduced in [8] as an alternative way to solve the Gribov problem. Here, we present the replica model by starting with a scalar field with a quartic self-interaction term:

$$S = \int d^4x \left[\frac{1}{2} \Phi(-\partial^2 + m^2)\Phi + g \Phi^4 \right]. \quad (40)$$

Then, we define a replica of the action above,

$$S' = \int d^4x \left[\frac{1}{2} \Phi'(-\partial^2 + m^2)\Phi' + g \Phi'^4 \right], \quad (41)$$

with the same parameters m and g . The two theories interact by a soft term depending on a free parameter, say μ^2 , and the replica model is then written as

$$S_{\text{replica}} = \int d^4x \left[\frac{1}{2} \Phi(-\partial^2 + m^2)\Phi + \frac{1}{2} \Phi'(-\partial^2 + m^2)\Phi' + i\mu^2 \Phi\Phi' + g \Phi^4 + g \Phi'^4 \right]. \quad (42)$$

The replica model (42) enjoys a symmetry which guarantees the existence of an unique mass parameter m and an unique quartic coupling constant g for both sectors of the theory (the original starting point model and its replica). This symmetry is often called *mirror symmetry* and its given by:

$$\Phi \rightarrow \Phi', \quad \Phi' \rightarrow \Phi. \quad (43)$$

Notice also that, when $m^2 = 0$, the propagator of Φ field is of the Gribov type, with μ^2 playing the role of the Gribov parameter¹⁰. Thus, it can also be diagonalized in terms of i -particles. In the deep ultraviolet regime, the two theories completely decouple and we obtain that the Φ field is said deconfined exhibiting a Yukawa-like propagator. The μ^2 parameter might be fixed by a gap equation like in the GZ model (16).

3 Taking a closer look at the $SU(3)$ group

3.1 General considerations

The $SU(N)$ group is the group of the $N \times N$ unitary matrices with determinant equals to one:

$$SU(N) := \{ U \mid UU^\dagger = \mathbf{1}, \det(U) = 1 \}. \quad (44)$$

The matrices $U \in SU(N)$ can be written as

$$U(\omega) := \exp(i\omega^a T^a) \quad (45)$$

where ω^a is a parameter, the label a runs from 1 to $(N^2 - 1)$, and T^a are the generators of the group, obeying the following relations:

$$\begin{aligned} [T^a, T^b] &= if^{abc}T^c, \\ \{T^a, T^b\} &= \frac{1}{N} \delta^{ab} + d^{abc}T^c. \end{aligned} \quad (46)$$

In equation (46) $[,]$ stands for the commutator, while $\{ , \}$ for the anti-commutator; f^{abc} are structure constants, which are anti-symmetric by odd successive permutations, *i.e.*

$$f^{abc} = -f^{bac} = -f^{acb} = -f^{cba}; \quad (47)$$

and d^{abc} are the components of the completely symmetric invariant rank-3 tensor of the group.

¹⁰For $m^2 \neq 0$ the propagator behaves like the most general RGZ model [7].

3.2 The $SU(2)$ group

Before discuss the $SU(3)$ case it is useful to spend a few words on the $SU(2)$ case. In this case there are three generators and they are related with the Pauli matrices, σ^a , as follows:

$$T^a = \frac{\sigma^a}{2}, \quad (a = 1, 2, 3), \quad (48)$$

where,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (49)$$

Also, we have

$$f^{abc} = \varepsilon^{abc}, \quad d^{abc} = 0, \quad (50)$$

and than

$$\left[\frac{\sigma^a}{2}, \frac{\sigma^b}{2} \right] = i\varepsilon^{abc} \frac{\sigma^c}{2}, \quad \left\{ \frac{\sigma^a}{2}, \frac{\sigma^b}{2} \right\} = \frac{1}{2} \delta^{ab}. \quad (51)$$

3.3 The $SU(3)$ group

In the $SU(3)$ group there are eight generators associated with the Gell-Mann matrices¹¹, λ^a , in an analog way as the Pauli matrices for $SU(2)$,

$$T^a = \frac{\lambda^a}{2}, \quad (a = 1, \dots, 8). \quad (52)$$

The Lie algebra of the generators of $SU(3)$ is than given by

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right] = if^{abc} \frac{\lambda^c}{2}, \quad \left\{ \frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right\} = \frac{1}{3} \delta^{ab} + d^{abc} \frac{\lambda^c}{2}. \quad (53)$$

Here, the generators obey the anti-symmetric and symmetric Jacobi identities:

$$\begin{aligned} [\lambda^a, [\lambda^b, \lambda^c]] + [\lambda^c, [\lambda^a, \lambda^b]] + [\lambda^b, [\lambda^c, \lambda^a]] &= 0, \\ [\lambda^a, \{\lambda^b, \lambda^c\}] + [\lambda^c, \{\lambda^a, \lambda^b\}] + [\lambda^b, \{\lambda^c, \lambda^a\}] &= 0. \end{aligned}$$

These identities give rise to the following useful relations:

$$\begin{aligned} f^{abc} f^{cde} + f^{adc} f^{ceb} + f^{aec} f^{cbd} &= 0, \\ f^{ade} d^{dbc} + f^{cde} d^{dab} + f^{bde} d^{dca} &= 0. \end{aligned} \quad (54)$$

In particular, the second equation of (54) is satisfied for $SU(N > 3)$ and the first one is valid for any value of N . Another interesting feature of the $SU(3)$ group is that its generators can be grouped in sets which obeys the same algebraic properties of the Pauli matrices determining then three $SU(2)$ subalgebras. Defining

$$\lambda_{\pm} = \frac{1}{2}(\sqrt{3}\lambda_8 \pm \lambda_3), \quad (55)$$

the three $SU(2)$ groups embedded in the $SU(3)$ are given by:

$$\begin{aligned} SU(2)_{\text{I}} &: \left(\frac{\lambda_1}{2}, \frac{\lambda_2}{2}, \frac{\lambda_3}{2} \right), \\ SU(2)_{\text{II}} &: \left(\frac{\lambda_4}{2}, \frac{\lambda_5}{2}, \frac{\lambda_+}{2} \right), \\ SU(2)_{\text{III}} &: \left(\frac{\lambda_6}{2}, \frac{\lambda_7}{2}, \frac{\lambda_-}{2} \right). \end{aligned} \quad (56)$$

¹¹The Gell-Mann matrices and the structure constants of the $SU(3)$ can be found in Appendix B.

The gauge field A_μ can be expanded in a base of the $SU(3)$ generators as

$$A_\mu = \sum_{a=1}^8 A_\mu^a \frac{\lambda^a}{2} = \sum_{a \neq 3,8} A_\mu^a \frac{\lambda^a}{2} + A_\mu^+ \lambda_+ + A_\mu^- \lambda_- , \quad (57)$$

where

$$A_\mu^\pm = \frac{1}{2} \left(\frac{1}{\sqrt{3}} A_\mu^8 \pm A_\mu^3 \right) . \quad (58)$$

It is interesting to comment here that the pure A -field sector of the $SU(3)$ Yang-Mills action, *i.e.*

$$S_{A\text{-field}} = \frac{1}{4} \int d^4x F_{\mu\nu}^a F_{\mu\nu}^a , \quad (59)$$

is left invariant by the transformations¹²

$$\begin{aligned} (A_\mu^1, A_\mu^2) &\rightarrow (A_\mu^1, A_\mu^2) , \\ (A_\mu^+, A_\mu^4, A_\mu^5) &\rightarrow (-A_\mu^-, A_\mu^7, A_\mu^6) , \\ (A_\mu^-, A_\mu^6, A_\mu^7) &\rightarrow (-A_\mu^+, -A_\mu^5, -A_\mu^4) . \end{aligned} \quad (60)$$

Notice that these transformations interchange the groups $SU(2)_{\text{II}}$ and $SU(2)_{\text{III}}$ and it reminds us the mirror symmetry of the replica model described in the preceding section. However, we can not identify these two subsectors of the theory as a replica model, at least not as originally conceived in [8], because of the presence of hard interaction terms. Furthermore, it is important to anticipate here a result that we will use later. In next section we will introduce the operator $d^{abc} A_\mu^b A_\mu^c$ coupled to an external source J^a . A possible condensation of this operator will give rise to a non-zero expectation value in the vacuum of J^a .¹³ It implies that we have to choose a direction for J^a and the only direction that maintains the symmetry (60) is the direction 3. Then, choosing $\langle J^a \rangle = m^2 \delta^{a3}$ we have

$$i \langle J^a \rangle d^{abc} A_\mu^b A_\mu^c = \frac{im^2}{3} (A_\mu^+ A_\mu^+ - A_\mu^- A_\mu^-) + \frac{im^2}{2} (A_\mu^4 A_\mu^4 - A_\mu^7 A_\mu^7) + \frac{im^2}{2} (A_\mu^5 A_\mu^5 - A_\mu^6 A_\mu^6) . \quad (61)$$

4 The model: The $SU(3)$ Yang-Mills with dimension 2 condensates

We will begin with the pure Yang-Mills action in the Landau gauge, eq.(4). Taking this action into account and trying to do not change the fundamental behavior in the ultraviolet regime we limit ourselves to operators of ultraviolet dimension 2. These type of operators give rise to soft breaking terms. In particular, the mass operator $\frac{1}{2} A_\mu^a A_\mu^a$ is well understood in the context of the local composite operator method (LCO) [24] and is responsible for a Yukawa type propagator. We will focus into another dimension 2 operator. One that is only possible into $SU(N \geq 3)$, the operator constructed with the symmetric structure constant and the gauge field A_μ^a , *i.e.* $\frac{1}{2} d^{abc} A_\mu^b A_\mu^c$. At this point it is important to emphasize that although we introduce the operator consistently with the study of the renormalizability via a BRST doublet of sources. We will be interested in bringing the sources to their physical value, following the same procedure of Gribov-Zwanziger [25, 26]. It is perfectly possible the introduction of these two operators in the quantum action and study the renormalizability of both, it is done in Appendix A. In order to turn clear the importance of each operator and the relation of then to the two type of solutions coming from

¹²Of course that the symmetry can be extended by the ghost and gauge fixing sectors of action (4)

¹³We are not calculating the value of this condensate. We limit ourselves in this work to study the consequences of its existence.

the Schwinger-Dyson equations, the scaling type and the decoupling one, in section 3 we will discuss the relation of these solutions and the operators $\frac{1}{2}d^{abc}A_\mu^b A_\mu^c$ and $\frac{1}{2}A_\mu^a A_\mu^a$.

In order to introduce the operator $\frac{1}{2}d^{abc}A_\mu^b A_\mu^c$, it is also necessary to introduce a BRST doublet of sources

$$s\lambda^a = iJ^a, \quad sJ^a = 0. \quad (62)$$

Then, the BRST invariant action including the dimension 2 operator $\frac{1}{2}d^{abc}A_\mu^b A_\mu^c$ is given by:

$$\begin{aligned} \Sigma &= S_{\text{YM}} + s \int d^4x \left(\frac{1}{2} \lambda^a d^{abc} A_\mu^b A_\mu^c - \frac{i\varepsilon}{2} \lambda^a J^a + \alpha \lambda^a \partial_\mu A_\mu^a \right) \\ &= S_{\text{YM}} + \int d^4x \left(\frac{i}{2} J^a d^{abc} A_\mu^b A_\mu^c + \lambda^a d^{abc} (\partial_\mu c^b) A_\mu^c - \frac{g}{2} f^{abc} d^{cde} \lambda^a c^b A_\mu^d A_\mu^e + \frac{\varepsilon}{2} J^a J^a \right) \\ &\quad + \alpha \int d^4x \left(iJ^a \partial_\mu A_\mu^a + \lambda^a \partial_\mu D_\mu^{ab} c^b \right). \end{aligned} \quad (63)$$

The α terms are necessary by algebraic renormalization consistency, due to the fact that there is any symmetry to exclude such terms¹⁴. The source J^a can have a nonzero expectation value. In fact, setting $\langle J^a \rangle = m^2 \delta^{a3}$, the action (63) gives rise to a propagator of the form:

$$\begin{aligned} \langle A_\mu^a(k) A_\nu^b(-k) \rangle &= \left[\frac{1}{k^2} \sum_{i=1}^2 \delta^{ai} \delta^{bi} + \frac{1}{k^2 + i\frac{m^2}{2}} \sum_{i=4}^5 \delta^{ai} \delta^{bi} + \frac{1}{k^2 - i\frac{m^2}{2}} \sum_{i=6}^7 \delta^{ai} \delta^{bi} \right. \\ &\quad \left. + \frac{k^2}{k^4 + \frac{m^4}{3}} (\delta^{a8} \delta^{b8} + \delta^{a3} \delta^{b3}) - i\frac{m^2}{\sqrt{3}} \frac{1}{k^4 + \frac{m^4}{3}} (\delta^{a8} \delta^{b3} + \delta^{a3} \delta^{b8}) \right] \theta_{\mu\nu}(k), \end{aligned} \quad (64)$$

with $\theta_{\mu\nu}(k)$ being the transverse projector,

$$\theta_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right). \quad (65)$$

This calculation is done in the Gell-Mann representation [11], see Appendix B. Although explicitly breaking the group, this is a soft breaking and in the ultraviolet regime is expected that the group structure is recovered. It is important to stress here that in spite of have two massless poles in the propagator it is not necessary that we have physical particles directly associated to this propagator. The interaction terms will mix the *i-particles* with the massless ones. This property can be easily observed if we rewrite the propagator as:

$$\begin{aligned} \langle A_\mu^a(k) A_\nu^b(-k) \rangle &= \left\{ \frac{1}{k^4 + \frac{m^4}{4}} (k^2 \delta^{ab} - im^2 d^{ab3}) + \frac{m^4}{4(k^4 + \frac{m^4}{4})} \left[\frac{1}{k^2} \sum_{i=1}^2 \delta^{ai} \delta^{bi} \right. \right. \\ &\quad \left. \left. - \frac{1}{3} \frac{k^2}{k^4 + \frac{m^4}{3}} (\delta^{a8} \delta^{b8} + \delta^{a3} \delta^{b3}) + i\frac{m^2}{3\sqrt{3}} \frac{1}{k^4 + \frac{m^4}{3}} (\delta^{a8} \delta^{b3} + \delta^{a3} \delta^{b8}) \right] \right\} \theta_{\mu\nu}(k), \end{aligned} \quad (66)$$

where d^{ab3} are symmetric structure constants in the 3 direction. In particular, for further use, we also display:

$$\langle A_\mu^1(k) A_\nu^1(-k) \rangle = \langle A_\mu^2(k) A_\nu^2(-k) \rangle = \frac{1}{k^2} \theta_{\mu\nu}(k),$$

¹⁴Also, in Landau gauge these extra terms can be absorbed by performing a linear shift of the fields variables b^a and \bar{c}^a :

$$b^a \rightarrow b^a + i\alpha J^a, \quad \bar{c}^a \rightarrow \bar{c}^a + i\alpha \lambda^a.$$

$$\begin{aligned}
\langle A_\mu^4(k) A_\nu^4(-k) \rangle &= \langle A_\mu^5(k) A_\nu^5(-k) \rangle = \frac{1}{k^2 + i\frac{m^2}{2}} \theta_{\mu\nu}(k), \\
\langle A_\mu^6(k) A_\nu^6(-k) \rangle &= \langle A_\mu^7(k) A_\nu^7(-k) \rangle = \frac{1}{k^2 - i\frac{m^2}{2}} \theta_{\mu\nu}(k), \\
\langle A_\mu^3(k) A_\nu^3(-k) \rangle &= \langle A_\mu^8(k) A_\nu^8(-k) \rangle = \frac{k^2}{k^4 + \frac{m^4}{3}} \theta_{\mu\nu}(k), \\
\langle A_\mu^3(k) A_\nu^8(-k) \rangle &= \langle A_\mu^8(k) A_\nu^3(-k) \rangle = -i \frac{m^2}{\sqrt{3}} \frac{1}{k^4 + \frac{m^4}{3}} \theta_{\mu\nu}(k).
\end{aligned} \tag{67}$$

At this point some considerations about the explicitly breaking of the $SU(3)$ are necessary. First, as was presented in section 3.3, the Gell-Mann matrices grouped as in (56) have the same algebraic properties as the Pauli matrices and so determine three natural $SU(2)$ subalgebras. So taking

$$h_1 = \lambda^3, \quad h_2 = \lambda^8, \quad e_\pm^1 = \lambda^1 \pm i\lambda^2, \quad e_\pm^2 = \lambda^6 \pm i\lambda^7, \quad e_\pm^3 = \lambda^4 \pm i\lambda^5,$$

it is easy to observe that the e_\pm^i obeys the following algebra

$$\begin{aligned}
[h_1, h_2] &= 0, \quad [\sqrt{3}h_2 + h_1, e_\pm^1] = \pm e_\pm^1, \quad [\sqrt{3}h_2 + h_1, e_\pm^2] = \mp \frac{1}{2} e_\pm^2, \quad [\sqrt{3}h_2 + h_1, e_\pm^3] = \pm \frac{1}{2} e_\pm^3, \\
[\sqrt{3}h_2 - h_1, e_\pm^1] &= \mp e_\pm^1, \quad [\sqrt{3}h_2 - h_1, e_\pm^2] = \pm \frac{1}{2} e_\pm^2, \quad [\sqrt{3}h_2 - h_1, e_\pm^3] = \mp \frac{1}{2} e_\pm^3, \\
[e_+^1, e_-^1] &= 2h_1, \quad [e_+^2, e_-^2] = \sqrt{3}h_2 - h_1, \quad [e_+^3, e_-^3] = \sqrt{3}h_2 + h_1.
\end{aligned} \tag{68}$$

This give rise to the well know weight diagram [11]. Moreover, from the properties of $SU(2)$ representations we know that $2p = m_1; \sqrt{3}q - p = m_2; \sqrt{3}q + p = m_3$, where (p, q) corresponds respectively to the eigenvalues of h_1 and h_2 ordered as points in \mathbb{R}^2 . Assuming that the trace of the propagator must be real, the natural choice of the direction of the breaking in h_1 give rise to two goldstone bosons associated to e_\pm^1 . The remaining 2 sets of $SU(2)$ corresponds to a similar structure as the replica model [8]. It is important to stress here that this is not a model for confinement in $SU(3)$. This is essentially an alternative mechanism for Gribov that presents confinement in the remaining two $SU(2)$ groups and has some defined observables associated to the remaining $SU(2)$ groups. In section 6 we will return to these issues in order to define the relationship between observables and the remaining group structure.

5 Taking into account the Gribov copies. The scaling type solution for the gluon and ghost propagators

In order to offer a better understanding of the model, a dynamical framework for the parameter $\langle J^a \rangle = m^2 \delta^{a3}$ should be provided, *i.e.* we should be able to establish a gap equation for that parameter as in the Gribov approximation [4] or in the GZ theory [5, 26]. The most immediate way to achieve a meaningful gap equation for m^2 is following the steps detailed in (9)→(16), also is relevant to remember that $d^{aac} \langle J^c \rangle = 0$ ¹⁵. The no-pole condition is implemented by stating that

$$\sigma(0, A) = 1, \tag{69}$$

with the propagator (64), which yields the gap equation determining the parameter m^4 , or equivalently $\langle J^a \rangle \langle J^a \rangle$, where dimensional regularization, $D = 4 - \epsilon$, has been employed. Therefore, at one-loop order,

¹⁵This general property can be easily seen, for example, in the Gell-Mann representation for the generators.

the gap equation yields

$$1 = g^2 \frac{3}{4} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^4 + \frac{m^4}{4}} + g^2 \frac{1}{16} \int \frac{d^D k}{(2\pi)^D} \frac{m^4}{k^2} \left(\frac{1}{k^4 + \frac{m^4}{4}} \right) \left(\frac{1}{k^4 + \frac{m^4}{3}} \right), \quad (70)$$

which enables us to express the parameter m^4 as a function of the coupling constant g . It is clear that the second integral is absolutely convergent and do not change the fact that m is determined as a function of a regularization mass Λ . Also from the gap equation (69) it follows that, for small values of the momentum

$$\sigma(k^2, m^4) \Big|_{k^2 \approx 0} = 1 - Ck^2, \quad (71)$$

and we recover the ghost propagator which is enhanced in the infrared $\langle \bar{c}^a(k) c^b(-k) \rangle \approx \frac{1}{k^4}$. We have thus recovered the so-called scaling solution, *i.e.* a suppressed gluon propagator which vanishes at the origin, and enhanced ghosts, which corresponds to the solution of the GZ theory.

5.1 Taking into account $\frac{1}{2} A_\mu^a A_\mu^a$ and the decoupling type solution.

Recent lattice numerical simulations [27, 28, 29, 30, 31, 33, 32] indicates a gluon propagator which is suppressed in the infrared and which attains a finite non-vanishing value at zero momentum, while the ghost propagator turns out to be not enhanced, *i.e.* $\langle \bar{c}^a(k) c^b(-k) \rangle \approx \frac{1}{k^2}$. This behaviour is known as the decoupling solution and has also been obtained from the analysis of the Schwinger-Dyson equations [34, 35, 36]. This solution appears in the Gribov-Zwanziger theory when the dynamics of the localizing fields is taken into account. In our model this behaviour is associated to the mass operator $\frac{1}{2} A_\mu^a A_\mu^a$. Before presenting the equations that characterize the stability of this operator it is important to emphasize that the tad pole presented in (104) despite being ultraviolet convergent has problems in infrared, but not necessarily at $k = 0$. The best chance to solve this divergency is introducing a mass term in the action.

The action Σ with a mass term is given by:

$$\Sigma_\mu = \Sigma + \frac{\mu^2}{2} \int d^4 x A_\mu^a A_\mu^a, \quad (72)$$

where Σ was defined in (63). The BRST variation of the mass term, turns out to be proportional to the equation of motion of b^a *i.e.*

$$\mathcal{S}(\Sigma_\mu) = 0 - \mu^2 \int d^4 x (\partial_\mu c^a) A_\mu^a = -i\mu^2 \int d^4 x c^a \frac{\delta \Sigma_\mu}{\delta b^a}, \quad (73)$$

modifying the Slavnov-Taylor to

$$\begin{aligned} \overline{\mathcal{S}}(\Sigma_\mu) &= \int d^4 x \left\{ \frac{\delta \Sigma_\mu}{\delta \Omega_\mu^a} \frac{\delta \Sigma_\mu}{\delta A_\mu^a} + \frac{\delta \Sigma_\mu}{\delta L^a} \frac{\delta \Sigma_\mu}{\delta c^a} + i b^a \frac{\delta \Sigma_\mu}{\delta \bar{c}^a} + i j^a \frac{\delta \Sigma_\mu}{\delta \lambda^a} + i \mu^2 c^a \frac{\delta \Sigma_\mu}{\delta b^a} \right\} \\ &= 0. \end{aligned} \quad (74)$$

It is now clear that the gauge propagator changes to a propagator in close relation to the one obtained from the refined Gribov-Zwanziger theory [37, 7, 38]. The new propagator is given by:

$$\begin{aligned} \langle A_\mu^a(k) A_\nu^b(-k) \rangle &= \left[\left(\frac{1}{(k^2 + \mu^2)^2 + \frac{m^4}{4}} \right) ((k^2 + \mu^2) \delta^{ab} - i m^2 d^{ab3}) \right. \\ &+ \frac{m^4}{4((k^2 + \mu^2)^2 + \frac{m^4}{4})} \left\{ \left(\frac{1}{k^2 + \mu^2} \right) \sum_{i=1}^2 \delta^{ai} \delta^{bi} - \frac{1}{3} \left(\frac{k^2 + \mu^2}{(k^2 + \mu^2)^2 + \frac{m^4}{3}} \right) (\delta^{a8} \delta^{b8} + \delta^{a3} \delta^{b3}) \right. \\ &+ \left. \left. i \frac{m^2}{\sqrt{3}} \frac{1}{3} \left(\frac{1}{(k^2 + \mu^2)^2 + \frac{m^4}{3}} \right) (\delta^{a8} \delta^{b3} + \delta^{a3} \delta^{b8}) \right\} \right] \theta_{\mu\nu}, \end{aligned} \quad (75)$$

with m^4 defined in (102). This expression has a finite nonvanishing value at zero momentum characterizing the decoupling type solution. It is important to emphasize here that lattice simulation results that have obtained the behaviour of the propagator are making use of the trace of the propagator. Remembering that $d^{aab} = 0$, the dominating term that lattice is capable to see is $\frac{k^2 + \mu^2}{k^4 + 2\mu^2 k^2 + \frac{\mu^4}{4} + \mu^4}$. It is clear that the dynamical origem of this mass parameter needs more explanation and we pretend to do this into a future work.

6 Local composite operator and the Källén-Lehmann spectral representation

Remove the gauge fields of the physical spectrum of the theory is not enough for a model that attempts to offer an alternative, at least in part, to the original Gribov question. It is also necessary to present a candidate for physical observable that displays the Källén-Lehmann spectral representation and corresponds to an invariant composite operator. It is pointed out in [9, 8] that a local composite operator constructed with i -particles [9] has one-loop correlation function that exhibits the Källén-Lehmann spectral representation, i.e.

$$\mathcal{I}(p^2) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{((p-k)^2 + i\sqrt{2}\nu^2)(k^2 - i\sqrt{2}\nu^2)}, \quad (76)$$

exhibits a spectral representation, as:

$$\mathcal{I}(p^2) - \mathcal{I}(0) = \int_{2\sqrt{2}\nu^2}^{\infty} dv \rho(v) \left\{ \frac{1}{v + p^2} - \frac{1}{v} \right\}, \quad (77)$$

where the spectral density $\rho(v) = \frac{1}{16\pi^2} \frac{\sqrt{v^2 - 8\nu^4}}{v}$ is positive in the range of integration. This property help us to find an operator that has the desired analyticity properties. Now let us return to the breaking of the $SU(3)$ group in order to understand the mechanism that permits the existence of physical observables. First of all, one of the desired aspects for one observable is not only that has Källén-Lehmann spectral representation of a particle but also does not carries color index and be gauge invariant. It is clear that the last requirement is not possible due to the explicitly breaking of the BRST symmetry. This is also a problem in the original Gribov-Zwanziger due to the same problem. i.e. the explicitly symmetry breaking. In our case another question emerges, the explicitly breaking of the group structure. Fortunately the last question will be the answer in order to obtain observables that does not carry color index. At least of the remaining group structure.

Analyzing the i -particles concept presented in [9, 8] we can see that it is impossible to obtain from a Gribov type propagator an observable that do not corresponds to an integral of the type (76). This does not only suggests that the observables must be constructed taking into account i -particle type correlators in order to obtain a real particle pole, but also indicates that a mechanism in order to do that must mix two different types of particles as done in [8]. The natural candidates that emerges as possible observables can be associated to the remaining 2 sets of $\mathcal{L}(SU(2))$, in particular it is convenient to define the quantities $E_{\mu\nu}^{3+} \equiv \frac{1}{\sqrt{2}}(F_{\mu\nu}^4 + iF_{\mu\nu}^5)$, $E_{\mu\nu}^{3-} \equiv \frac{1}{\sqrt{2}}(F_{\mu\nu}^4 - iF_{\mu\nu}^5)$, $E_{\mu\nu}^{2+} \equiv \frac{1}{\sqrt{2}}(F_{\mu\nu}^6 + iF_{\mu\nu}^7)$ and $E_{\mu\nu}^{2-} \equiv \frac{1}{\sqrt{2}}(F_{\mu\nu}^6 - iF_{\mu\nu}^7)$ that obeys the same algebra as presented in (68). From these operators it is convenient to define one possible candidate to observable as:

$$\begin{aligned} \phi &\equiv E_{\mu\nu}^{2+} E_{\mu\nu}^{3+} + E_{\mu\nu}^{3-} E_{\mu\nu}^{2-} \\ \phi &= F_{\mu\nu}^4 F_{\mu\nu}^6 - F_{\mu\nu}^5 F_{\mu\nu}^7. \end{aligned} \quad (78)$$

It should be noted that in spite of the breaking of the BRST symmetry, the candidate to observable must be a singlet. It is important to remember that the $E_{\mu\nu}$ essentially obeys the same group properties as

defined in (68). Remenbering that we are performin the breaking in diretion 3 or in the notation presented in (68) h_1 , the algebra between h_1 and e_{\pm}^2, e_{\pm}^2 is given by:

$$[h_1, e_{\pm}^2] = \mp \frac{1}{2} e_{\pm}^2 \quad [h_1, e_{\pm}^3] = \pm \frac{1}{2} e_{\pm}^3, \quad (79)$$

making the algebraic result $[h_1, e_{\pm}^2 e_{\pm}^3 + e_{\pm}^3 e_{\pm}^2] = 0$ easily to obtain and proving that this operator is invariant under the remaining group charge defined by the algebra (79) Taking into account the propagators as:

$$\begin{aligned} \langle A_{\mu}^1(k) A_{\nu}^1(-k) \rangle &= \langle A_{\mu}^2(k) A_{\nu}^2(-k) \rangle = \frac{1}{k^2} \theta_{\mu\nu}(k) \\ \langle A_{\mu}^4(k) A_{\nu}^4(-k) \rangle &= \langle A_{\mu}^5(k) A_{\nu}^5(-k) \rangle = \frac{1}{k^2 + i \frac{m^2}{2}} \theta_{\mu\nu}(k) \\ \langle A_{\mu}^6(k) A_{\nu}^6(-k) \rangle &= \langle A_{\mu}^7(k) A_{\nu}^7(-k) \rangle = \frac{1}{k^2 - i \frac{m^2}{2}} \theta_{\mu\nu}(k) \\ \langle A_{\mu}^3(k) A_{\nu}^3(-k) \rangle &= \langle A_{\mu}^8(k) A_{\nu}^8(-k) \rangle = \frac{k^2}{k^4 + \frac{m^4}{3}} \theta_{\mu\nu}(k) \\ \langle A_{\mu}^3(k) A_{\nu}^8(-k) \rangle &= \langle A_{\mu}^8(k) A_{\nu}^3(-k) \rangle = -i \frac{m^2}{\sqrt{3}} \frac{1}{k^4 + \frac{m^4}{3}} \theta_{\mu\nu}(k), \end{aligned} \quad (80)$$

the first candidate which has the desired one-loop correlation function [9] is given by:

$$\begin{aligned} \langle \phi(k) \phi(-k) \rangle &= 12 \int_{m^2}^{\infty} dv \frac{\rho(v)}{v + k^2}, \\ \rho(v) &= \frac{\sqrt{v^2 - m^4} (v^2 + m^4)}{\pi^2 v}. \end{aligned} \quad (81)$$

There is another observable with different value for the mass of the condensate. In order to understand the second candidate to an observable, i.e. presents the Källén-Lehmann spectral representation of a particle, it is convenient to consider the quadratic part of the action, in particular the term with A_{μ}^3 and A_{μ}^8 .

$$S_{3,8} = \int d^4x \left\{ \frac{1}{2} A_{\mu}^3(-\partial^2) A_{\mu}^3 + \frac{1}{2} A_{\mu}^8(-\partial^2) A_{\mu}^8 + m^2 \frac{i}{\sqrt{3}} A_{\mu}^3 A_{\mu}^8 \right\}, \quad (82)$$

where we have already taken into account the Landau gauge conditions, $\partial_{\mu} A_{\mu}^3 = 0$ and $\partial_{\mu} A_{\mu}^8 = 0$. This sector of the action can be diagonalized trivially making the field redefinition.

$$\begin{aligned} U_{\mu} &= \frac{1}{\sqrt{2}} (A_{\mu}^3 + A_{\mu}^8) \\ V_{\mu} &= \frac{1}{\sqrt{2}} (-A_{\mu}^3 + A_{\mu}^8). \end{aligned} \quad (83)$$

Therefore

$$S_{3,8} = \int d^4x \left\{ \frac{1}{2} U_{\mu} (-\partial^2 + i \frac{m^2}{\sqrt{3}}) U_{\mu} + \frac{1}{2} V_{\mu} (-\partial^2 - i \frac{m^2}{\sqrt{3}}) V_{\mu} \right\}, \quad (84)$$

which describes again the *i-particle* structure. In order to write a physical operator that has the desired structure, it is sufficient to study the operator

$$\chi = (\partial_{\mu} U_{\nu} - \partial_{\nu} U_{\mu}) (\partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu}), \quad (85)$$

where its two-point correlation function can be cast in the form of a Källén-Lehmann spectral representation of a physical particle,

$$\begin{aligned} \langle \chi(k)\chi(-k) \rangle &= \frac{3}{8} \int_{2\frac{m^2}{\sqrt{3}}}^{\infty} dv \frac{\rho(v)}{v+k^2}, \\ \rho(v) &= \frac{\sqrt{v^2 - \frac{4}{3}m^4}(v^2 + \frac{4}{3}m^4)}{\pi^2 v}. \end{aligned} \quad (86)$$

Again it is convenient to observe that this operator can be associated to the abelian subgroup of the original $SU(3)$ group and it is clear that the mass pole only depends on the mass gap of the *i-particle*. Again it is important to stress that due to the fact that the insertion is a soft broken, the group symmetry is recovered at the limit $k \rightarrow \infty$. Here is important to comment that a dynamical symmetry breaking could be the answer to a BRST invariante observable. In this case the full BRST operator carries not only the gauge fields but also the auxiliary fields. In this case it could be possible to define a BRST and group invariant colorless physical observable.

7 Conclusions

In this work we have studied the $SU(3)$ Yang-Mills theory in a Landau gauge with a soft mass term proportional to the symmetric structure constant. This soft mass term allow us to treat the Gribov problem in a local renormalizable action without the need of auxiliary fields like in the Gribov-Zwanziger theory. The confining behavior is thus induced by the soft mass term and the relation with the scaling solution is presented in close analogy to the Gribov mechanism and to the replica model[8]. Also the introduction of the diagonal mass term and the relation with the decoupling solution is discussed. Moreover this can open the possibility of a more general relation between confining behavior and the existence of gauge condensates in the infrared regime. In fact, it would be interesting the possibility of obtaining the value of the mass gap by the local composite operator method and study the relation between the extremum of the effective potential and the Gribov mass gap equation.

The result we have obtained suggests further that the restriction to the first Gribov region could be implemented into other gauges by a simple soft mass term involving the symmetric structure constant that breaks the original group into a more simple group structure with a replica. Further studies aimed at establishing such a connection, in particular for the maximal abelian gauge, in a more precise way is called for and is a topic of current investigation.

Another important point analysed is the physical operators associated to these propagator. Due to the structure of the propagator this is a difficult task. The simple *i-particle* structure is not present in these model and the mechanism associated to the *i-particle* is much more complex and involves the group structure itself. This is the price to be paid in order to do not double the number of gauge fields or introduce auxiliary localizing fields. Nevertheless we find possible candidates of physical operators associated to this propagator in which the group structure is present in the ultraviolet regime. It is argued that a dynamical breaking mechanism could restore the full BRST invariance of the observable, in particular for $F_{\mu\nu}^a(x)F_{\mu\nu}^a(x)$ in the target remaining group structure. The construction of all this mechanism for a target remaining group like an $SU(3)$ replica is another topic to be investigated into future works.

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A Renormalizability

A.1 Equations Compatible with the Quantum Action Principle

The full set of equations compatible with the quantum action principle [12] is given by:

- The Lagrange multiplier and the antighost equation.

$$\begin{aligned}\frac{\delta\Sigma}{\delta b^a} &= i\partial_\mu A_\mu^a \\ \frac{\delta\Sigma}{\delta \bar{c}^a} + \partial_\mu \frac{\delta\Sigma}{\delta \Omega_\mu^a} &= 0\end{aligned}\tag{87}$$

- The integrated ghost equation

$$\begin{aligned}\mathcal{G}^a(\Sigma) &= \Delta^a \\ \mathcal{G}^a(\Sigma) &= \int d^4x \left\{ \frac{\delta\Sigma}{\delta c^a} - igf^{abc}(\bar{c}^b \frac{\delta\Sigma}{\delta b^c} + \lambda^b \frac{\delta\Sigma}{\delta j^c}) \right\}, \\ \Delta^a &= \int d^4x \{ gf^{abc}(\Omega_\mu^b + \alpha\partial_\mu \lambda^b)A_\mu^c - gf^{abc}(L^b c^c + i\varepsilon\lambda^b j^c) \}.\end{aligned}\tag{88}$$

Before presenting all the equations compatible with the quantum action principle it is relevant to note here that the term $\frac{\varepsilon}{2}j^a j^a$ generates a linear breaking in the ghost equation. This will give us the information that this term does not renormalizes.

- Slavnov-Taylor

$$\begin{aligned}\mathcal{S}(\Sigma) &= \int d^4x \left\{ \frac{\delta\Sigma}{\delta \Omega_\mu^a} \frac{\delta\Sigma}{\delta A_\mu^a} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta\Sigma}{\delta c^a} + ib^a \frac{\delta\Sigma}{\delta \bar{c}^a} + ij^a \frac{\delta\Sigma}{\delta \lambda^a} \right\} \\ \mathcal{S}(\Sigma) &= 0.\end{aligned}\tag{89}$$

- Rigid symmetry

$$\begin{aligned}W^a(\Sigma) &= \int d^4x gf^{abc} \left\{ A_\mu^b \frac{\delta\Sigma}{\delta A_\mu^c} + c^b \frac{\delta\Sigma}{\delta c^c} + b^b \frac{\delta\Sigma}{\delta b^c} + \bar{c}^b \frac{\delta\Sigma}{\delta \bar{c}^c} + \Omega_\mu^b \frac{\delta\Sigma}{\delta \Omega_\mu^c} + L^b \frac{\delta\Sigma}{\delta L^c} + \lambda^b \frac{\delta\Sigma}{\delta \lambda^c} + j^b \frac{\delta\Sigma}{\delta j^c} \right\} \\ W^a(\Sigma) &= 0.\end{aligned}\tag{90}$$

- $SL(2, R)$

$$R(\Sigma) = \int d^4x \left\{ c^a \frac{\delta\Sigma}{\delta \bar{c}^a} - i \frac{\delta\Sigma}{\delta b^{ca}} \frac{\delta\Sigma}{\delta L^a} \right\} = 0.\tag{91}$$

A.2 Stability of the quantum action

The next step is to characterize the most general counterterm that can be freely added to all orders in perturbation theory respecting all the symmetries presented previously. Following the set up of the Algebraic Renormalization [12], we perturb the classical action Σ by adding an integrated local polynomial in the fields and sources, Σ_{count} , with dimension bounded by four, and with vanishing ghost number. The perturbed action $(\Sigma + \epsilon\Sigma_{count})$, where ϵ is an expansion parameter, fulfills, to the first order in ϵ , the same Ward identities obeyed by the classical action Σ , *i.e* equations (87)-(91).

$$\begin{aligned}\beta_\Sigma \Sigma_{count} &= 0, & \frac{\delta\Sigma_{count}}{\delta b^a} &= 0, & (\frac{\delta}{\delta \bar{c}^a} + \partial_\mu \frac{\delta}{\delta \Omega_\mu^a}) \Sigma_{count} &= 0, \\ \mathcal{G}^a \Sigma_{count} &= 0, & W^a \Sigma_{count} &= 0, & R \Sigma_{count} &= 0,\end{aligned}\tag{92}$$

where β_Σ is given by:

$$\beta_\Sigma = \int d^4x \left\{ \frac{\delta\Sigma}{\delta\Omega_\mu^a} \frac{\delta}{\delta A_\mu^a} + \frac{\delta\Sigma}{\delta A_\mu^a} \frac{\delta}{\delta\Omega_\mu^a} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a} \frac{\delta}{\delta L^a} + ib^a \frac{\delta}{\delta \bar{c}^a} + ij^a \frac{\delta}{\delta \lambda^a} \right\} \quad (93)$$

thus, taking into account the general results on the cohomology of Yang-Mills theories, the most general invariant counterterm is:

$$\Sigma_{count} = \int d^4x \left\{ \frac{a_0}{4} F_{\mu\nu}^a F_{\mu\nu}^a \right\} + \beta_\Sigma \Delta^{-1}. \quad (94)$$

The integrated term corresponds to the nontrivial part of the cohomology of β_Σ , while Δ^{-1} is an integrated polynomial in the fields and sources with ultraviolet dimension 4 and ghost number -1. The ultraviolet dimension and ghost number of all fields and sources.

fields and sources	A_μ^a	c^a	\bar{c}^a	b^a	λ^a	j^a	Ω_μ^a	L^a
UV dimension	1	0	2	2	2	2	3	4
Ghost number	0	1	-1	0	-1	0	-1	-2

Table 1: Quantum numbers of fields and sources.

Applying all the constraints given in (92) and observing the ultraviolet dimension 4 and ghost number -1, we obtain for Δ^{-1} :

$$\Delta^{-1} = \int d^4x \left\{ a_1 (\Omega_\mu^a + \partial_\mu \bar{c}^a) A_\mu^a + a_2 \alpha \partial_\mu \lambda^a A_\mu^a + \frac{a_3}{2} \lambda^a d^{abc} A_\mu^b A_\mu^c \right\} \quad (95)$$

We see thus that Σ_{count} contains 4 free independent parameters, namely (a_0, a_1, a_2, a_3) . These parameters can be reabsorbed by means of a multiplicative renormalization of the gauge coupling constant g , off the parameters (α, ε) and the set of fields and sources $\phi = (A_\mu^a, c^a, \bar{c}^a, b^a, \Omega_\mu^a, L^a, \lambda^a, j^a)$ according to

$$\Sigma(g, \alpha, \phi) + \hbar \Sigma^{count} = \Sigma(g_0, \alpha_0, \phi_0) + O(\hbar^2), \quad (96)$$

with

$$\begin{aligned} g_0 &= Z_g g, & \alpha_0 &= Z_\alpha \alpha, & \varepsilon_0 &= Z_\varepsilon \varepsilon \\ A_{0\mu}^a &= Z_A^{\frac{1}{2}} A_\mu^a, & c_0^a &= Z_c c^a, & b_0^a &= Z_b b^a, & \bar{c}_0^a &= Z_{\bar{c}} \bar{c}^a, \\ \lambda_0^a &= Z_\lambda \lambda^a, & j_0^a &= Z_j j^a, \end{aligned} \quad (97)$$

and

$$\begin{aligned} Z_g &= 1 - \hbar \frac{a_0}{2}, & Z_\alpha &= 1 + \hbar \left(\frac{a_0}{2} - a_2 - a_3 \right), & Z_\varepsilon &= 1 + \hbar (2a_0 - 2a_3), \\ Z_A^{\frac{1}{2}} &= 1 + \frac{\hbar}{2} (a_0 + 2a_1), & Z_c &= Z_{\bar{c}} = Z_\Omega = 1 - \hbar \frac{a_1}{2}, & Z_b &= 1 - \frac{\hbar}{2} (a_0 + 2a_1), \\ Z_\lambda &= 1 - \hbar \left(\frac{a_0}{2} - \frac{a_1}{2} - a_3 \right), & Z_j &= 1 - \hbar (a_0 - a_3), \end{aligned} \quad (98)$$

or directly in terms of the multiplicative relations between the renormalization factors Z

$$Z_b Z_A^{\frac{1}{2}} = 1, \quad Z_{\bar{c}} Z_g Z_A^{\frac{1}{2}} Z_c = 1, \quad Z_\varepsilon Z_j^2 = 1. \quad (99)$$

In order to clarify the importance of such relations and the Gribov type propagator, let us follow the Zwanziger prescription and set the sources λ^a, j^a respectively to 0, $\langle j^a \rangle$ with $\langle j^a \rangle \neq 0$. Taking this

into account it is clear that the action is BRST invariant up to a soft breacking term proportional to $\langle j^a \rangle$

$$s\Sigma(A_\mu^a, \bar{c}^a, c^a, b^a, \lambda^a = 0, L^a = 0, \Omega_\mu^a = 0, j^a = \langle j^a \rangle) = \langle j^a \rangle \Delta^a$$

$$\Delta^a = \int d^4x \{ d^{abc} \partial_\mu c^b A_\mu^c - \frac{g}{2} f^{abc} c^b d^{cde} A_\mu^d A_\mu^e + \alpha \partial_\mu D_\mu^{ab} c^b \}, \quad (100)$$

just like in the Zwanziger procedure to the Gribov problem and due to the fact that we dont have localizing fields, the limit $\langle j^a \rangle \rightarrow 0$ clearly recover the pure Yang-Mills in the deep ultraviolet region.

A.3 Nonrenormalization of the soft mass term, simple one loop prove

Following closely the arguments presented by Sorella in [8] we will prove that there is no one loop correction to the soft mass term. The argument is based on dimensional regularization with minimal subtraction and the fact that, at least at one loop, the tadpole diagram in the two point $A-A$ is related to an integral of the type.

$$\int \frac{d^D x}{(2\pi)^D} f^{adc} f^{bec} \langle A_\mu^d(k) A_\nu^e(k) \rangle, \quad D = 4 - \epsilon \quad (101)$$

which can be rewritten as

$$\begin{aligned} \int \frac{d^D x}{(2\pi)^D} \{ f^{adc} f^{bdc} \left(\frac{k^2}{k^4 + \frac{m^4}{4}} \right) - i f^{adc} f^{bec} d^{de3} m^2 \left(\frac{1}{k^4 + \frac{m^4}{4}} \right) \\ + f^{adc} f^{bec} \frac{m^4}{4(k^4 + \frac{m^4}{4})} \left(\left(\frac{1}{k^2} \right) \sum_{i=1}^2 \delta^{di} \delta^{ei} - \frac{1}{3} \left(\frac{k^2}{k^4 + \frac{m^4}{3}} \right) (\delta^{d8} \delta^{e8} + \delta^{d3} \delta^{e3}) \right) \\ + i \frac{m^2}{\sqrt{3}} \frac{1}{3} \left(\frac{1}{k^4 + \frac{m^4}{3}} \right) (\delta^{d8} \delta^{e3} + \delta^{d3} \delta^{e8}) \} \end{aligned} \quad (102)$$

Now in order to prove that all terms do not contribute to the mass, at least in first order of perturbation theory, let us analyse the different terms of this integral. The first term give rise to:

$$\int \frac{d^D x}{(2\pi)^D} \frac{k^2}{k^4 + \mu^4} = \int \frac{d^D x}{(2\pi)^D} \frac{1}{k^2} - \int \frac{d^D x}{(2\pi)^D} \frac{\mu^4}{k^2(k^4 + \mu^4)}, \quad (103)$$

corresponding respectively to an integral that is zero by dimensional and a power counting ultraviolet convergent one. The second term to be analysed is of the form:

$$\int \frac{d^D x}{(2\pi)^D} \frac{1}{k^4 + \mu^4} = \int \frac{d^D x}{(2\pi)^D} \frac{1}{k^4} - \int \frac{d^D x}{(2\pi)^D} \frac{\mu^4}{k^4(k^4 + \mu^4)}. \quad (104)$$

Recursively the same argument is applied to the third integral, showing thus that no divergent terms proportional to $\langle j^3 \rangle$ arise. Also, from the absence of one-loop counterterm of the kind $\langle j^a \rangle$ $\frac{1}{2} d^{abc} A_\mu^b A_\mu^c$, it follows that

$$\langle j^a \rangle_0 \frac{1}{2} d^{abc} (A_\mu^b)_0 (A_\mu^c)_0 = \langle j^a \rangle \frac{1}{2} d^{abc} A_\mu^b A_\mu^c, \quad (105)$$

so that we obtain

$$\langle j^a \rangle_0 = Z_{\langle j \rangle} \langle j^a \rangle, \quad Z_{\langle j \rangle} Z_A = 1, \quad (106)$$

meaning that the renormalization factor of the soft parameter $\langle j^a \rangle$ can be expressed in terms of the gluon renormalization factor Z_A .¹⁶

¹⁶A purely algebraic proof, valid to all orders, of the non-renormalization properties of the soft parameter is under investigation. Also the possibility of obtaining the value of $\langle j^a \rangle$, by the local composite operator method[24, 39].

B The Gell-Mann matrices and the structure constants of $SU(3)$

The Gell-Mann matrices are given by

$$\begin{aligned}
\lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & \lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, & \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
\lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, & \lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.
\end{aligned} \tag{107}$$

The nonzero structure constants are:

$$f^{123} = 1, \quad f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2}. \tag{108}$$

The nonzero components of the symmetric tensor d^{abc} are:

$$\begin{aligned}
d^{118} &= d^{228} = d^{338} = -d^{888} = \frac{1}{\sqrt{3}}, \\
d^{448} &= d^{558} = d^{668} = d^{778} = -\frac{1}{2\sqrt{3}}, \\
d^{146} &= d^{157} = -d^{247} = d^{256} = d^{344} = d^{355} = -d^{366} = -d^{377} = \frac{1}{2}.
\end{aligned} \tag{109}$$